MINERALOGIA POLONICA
Vol. 8, No 2 — 1977

Józef Nedoma *

SIMPLIFIED NOTATION OF SYMMETRY OPERATIONS

UKD 548.12:512.831]:003.62

Abstract. A new simplified notation of symmetry operations is proposed. Simplified symbols of all point symmetry operations met with in crystallography can be easily *traduced* into corresponding matrices with aid of the generalized matrix of symmetry operations published in one of former papers. All known point symmetry operations are discussed from the view-point of simplified symbols.

INTRODUCTION

The generalized matrix derived previously (Nedoma 1976) describes all possible point symmetry operations coexisting with translation. To write a matrix representing a given symmetry operation five following values must be known:

α — the angle of rotation,

D — the value of the determinant (+1 for ordinary rotation axes, —1 for mirror axes),

M, N, P — coordinates of a point characterizing the position of the axis in space, the *M, N, P* — values fulfill the condition $M^2 + N^2 + P^2 = 1$ the axis passes trought the points *M, N, P* and 0, 0, 0.

In the foregoing paper an abbreviated symbol of the generalized matrix α (*D, M, N, P*) has been introduced. In the present paper the notation of this symbol will be further simplified.

THE SIMPLIFIED SYMBOL OF THE GENERALIZED MATRIX

The value of *D* appearing in the generalized matrix may assume only one of two values +1 and —1. The value of *D* must not be written explicitly in the simplified symbol. It is enough to assume two following symbols:

* Academy of Mining and Metallurgy, Institute of Material Science, Cracow (Kraków, al. Mickiewicza 30).

$$\alpha\,(M, N, P) \text{ for an axis with } D = 1$$

and

$$\underline{\alpha}\,(M, N, P) \text{ for an axis with } D = -1.$$

Instead of writing the rotation angle α in the symbol we can introduce the value

$$n = \frac{360}{\alpha}$$

making use of the fact that for point symmetry operations coexisting with translation the rotation angle α can assume only the values of 60, 90, 120, 180 and 360°.

The symbols 60 (M,N,P) 90 (M,N,P) 120 (M,N,P) 180 (M,N,P) and 360 (M,N,P) can be thus written shorter 6 (M,N,P) 4 (M,N,P) 3 (M,N,P) 2 (M,N,P) and 1 (M,N,P).

In the case of mirror axes the following analogous symbols can be used: $\underline{6}\,(M,N,P)$ $\underline{4}\,(M,N,P)$ $\underline{3}\,(M,N,P)$ $\underline{2}\,(M,N,P)$ $\underline{1}\,(M,N,P)$.

The values M, N, P appearing in the generalized matrix must fulfill the condition $M^2 + N^2 + P^2 = 1$. The values m, n, p appearing in the simplified symbol must not fulfill this condition, we must be only able to calculate the proper M, N, P values (needed for introduction into the generalized matrix) from the m, n, p values appearing in the symbol.

If the sum of squares of m, n, p-values appearing in the simplified symbol is equal to 1 we may assume $m = M$, $n = N$, $p = P$.

If this sum is equal to a value different from 1 (for instance A^2) the proper M, N, P — values can be easily calculated in the following way:

$$m^2 + n^2 + p^2 = A^2$$

$$\frac{m^2}{A^2} + \frac{n^2}{A^2} + \frac{p^2}{A^2} = 1$$

$$M = \frac{m}{A},\ N = \frac{n}{A},\ P = \frac{p}{A}.$$

The values m, n, p appearing in the simplified symbol and not fulfilling the condition $m^2 + n^2 + p^2 = 1$ — before introduction into the generalized matrix — must be then converted into the proper M, N, P — values. The conversion consists in dividing each of the m, n, p values by the square root of the sum of squares $m^2 + n^2 + p^2$.

Examples

1. The symbol $\underline{6}\,(0, 0, 1)$ — or simpler $\underline{6}\,(001)$ denotes the matrix of a sixfold mirror axis passing through points $0, 0, 0$ and $0, 0, 1$. The M, N, P — values to be introduced into the generalized matrix are $M = 0$, $N = 0$, $P = 1$ as the sum of m, n, p — squares is equal to 1.

2. The symbol $2\,(0, 1, 1)$ — or simples $2\,(011)$ denotes a matrix of a two-fold axis passing trough point $0, 1, 1$ and $0, 0, 0$. The sum of squares $m^2 + n^2 + p^2 = 2$ is not equal to 1: the M, N, P-values to be introduced into the generalized matrix must be therefore obtained by division by $\sqrt{2}$

$$M = 0 \qquad N = \frac{1}{\sqrt{2}} \qquad P = \frac{1}{\sqrt{2}}$$

3. The symbol $1\,(M,N,P)$ or simpler $1\,(MNP)$ denotes a rotation by 360°. Each point remains in its position, its coordinates do not change. The corresponding matrix

$$\begin{Vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{Vmatrix} = 1\,(MNP)$$

represents the transformation of identity and remains the same for all triples of MNP-values.

4. The symbol $\underline{2}\,(M,N,P)$ or $\underline{2}\,(MNP)$ denotes a rotation by 180° with a simultaneous reflection in a mirror plane perpendicular to the rotation axis and passing trough points $0, 0, 0$ and M, N, P. This operation is equivalent to a reflection in a symmetry center. The matrix

$$\begin{Vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{Vmatrix} = \underline{2}\,(MNP)$$

represents thus the reflection in a symmetry center and holds — similarly as $1\,(MNP)$ — for all possible triples of MNP-values.

5. The symbol $\underline{1}\,(M,N,P)$ denotes rotation by 360° around an axis passing trough the points M, N, P and $0, 0, 0$ with simultaneous reflection in a mirror plane perpendicular to this axis. As the rotation by 360° does not change the coordinates of points in space the operation consists in a reflection in a mirror plane only. The symbol $\underline{1}\,(M,N,P)$ denotes thus reflection in a mirror plane passing through the point $0, 0, 0$ perpendicularly to an axis passing trough the points M, N, P and $0, 0, 0$.

INVERSION AXES

Rotation by an angle α connected with simultaneous reflection in a symmetry center is called inverting rotation and the corresponding axis an inversion axis. In symbols introduced in this paper an inversion axis can be written as a result of following multiplication:

$$n\,(MNP) \cdot \underline{2}\,(MNP)$$

In matrix notation:

$$\begin{Vmatrix} M^2R + \cos\alpha & MNR - P\sin\alpha & MPR + N\sin\alpha \\ MNR + P\sin\alpha & N^2R + \cos\alpha & NPR - M\sin\alpha \\ MPR - N\sin\alpha & NPR + M\sin\alpha & P^2R + \cos\alpha \end{Vmatrix} \cdot \begin{Vmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{Vmatrix}$$

where: $R = 1 - \cos\alpha$

After multiplication we obtain for an inversion axis the following matrix

$$\begin{Vmatrix} M^2(-1+\cos\alpha)-\cos\alpha & MN(-1+\cos\alpha)+P\sin\alpha & MP(-1+\cos\alpha)-N\sin\alpha \\ MN(-1+\cos\alpha)-P\sin\alpha & N^2(-1+\cos\alpha)-\cos\alpha & NP(-1+\cos\alpha)+M\sin\alpha \\ MP(-1+\cos\alpha)+N\sin\alpha & NP(-1+\cos\alpha)-M\sin\alpha & P^2(-1+\cos\alpha)-\cos\alpha \end{Vmatrix}$$

The determinant calculated for a $n\,(MNP)$-axis is equal to 1, the determinant of the matrix $\underline{2}\,(MNP)$ is equal to -1. The determinant of the resulting matrix must be therefore equal to -1. In the resulting matrix the value of -1 is placed already on the right position for D. To transform this matrix into a general matrix of a $\underline{n}\,(MNP)$-operation we must change all signs of the MNP-values and introduce a new angle fulfilling the conditions

$$\cos\beta = -\cos\alpha \qquad \sin\beta = \sin\alpha$$

i.e.

$$\beta = 180 - \alpha$$

If we introduce for an inversion axis the symbol

$$\bar{\alpha}\,(MNP) \quad \text{or} \quad \bar{n}\,(MNP)$$

we can write generally

$$\bar{\alpha}\,MNP = \underline{180-\alpha}\,(-M,-N,-P)$$

or for given axes:

$$\bar{6}\,(M,N,P) = \underline{3}\,(-M,-N,-P)$$

$$\bar{4}\,(M,N,P) = \underline{4}\,(-M,-N,-P)$$

$$\bar{3}\,(M,N,P) = \underline{6}\,(-M,-N,-P)$$

$$\bar{2}\,(M,N,P) = \underline{1}\,(-M,-N,-P)$$

$$\bar{1}\,(M,N,P) = \underline{2}\,(M,N,P)$$

To write a matrix for an inversion axis we must transform the $\bar{n}$ into the proper $\underline{n}$ value, change all signs of M, N, P-values and introduce the new M, N, P-value and the new rotation angle into the generalized matrix.

We may also proceed in a different simpler way. Basing on the equation

$$\bar{n}\,(MNP) = n\,(MNP)\cdot\bar{1}\,(MNP)$$

we can write the matrix for the $n\,(MNP)$-axis and change in the resulting matrix the signs of all matrix elements into opposite ones.

Example

Let us write the matrix corresponding to the simplified symbol $\bar{6}\,(0,0,-1)$.

First way:

Transforming the inversion axis into the corresponding mirror axis we can write

$$\bar{6}\,(0,0,1) = \underline{3}\,(0,0,-1)$$

For $\underline{3}\,(0,0,-1)$ we have:

$$\alpha = 120^\circ, \qquad \cos\alpha = -\frac{1}{2},$$

$$D = -1, \qquad \sin\alpha = \frac{\sqrt{3}}{2},$$

$$R = -\frac{1}{2}.$$

After introduction into the generalized matrix we obtain

$$\begin{Vmatrix} -\frac{1}{2} & +\frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & -1 \end{Vmatrix} = \bar{6}\,(0,\ 0,\ 1)$$

Second way:

For $6\,(0,0,1)$ we have:

$$\alpha = 60, \qquad \cos\alpha = \frac{1}{2},$$

$$D = 1, \qquad \sin\alpha = \frac{\sqrt{3}}{2},$$

$$R = \frac{1}{2},$$

From the generalized matrix we obtain

$$\begin{Vmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{Vmatrix} = 6\,(0,\ 0,\ 1)$$

After having changed all signs of matrix elements multiplication by $\bar{1}$ (M, N, P) we obtain the former result.

Each point symmetry operation coexisting with translation can be thus described with one of following simplified symbols:

Rotation axes	Inversion axes	Mirror axes
$6\,(MNP)$	$\bar{6}\,(MNP)$	$= \underline{3}\,(-M,-N,-P)$
$4\,(MNP)$	$\bar{4}\,(MNP)$	$= \underline{4}\,(-M,-N,-P)$
$3\,(MNP)$	$\bar{3}\,(MNP)$	$= \underline{6}\,(-M,-N,-P)$
$2\,(MNP)$	$\bar{2}\,(MNP)$	$= \underline{1}\,(-M,-N,-P)$ mirror plane
$1\,(MNP)$ — indentity	$\bar{1}\,(MNP)$	$= \underline{2}\,(-M,-N,-P)$ symmetry center

REFERENCES

NEDOMA J., 1976: A generalized matrix of symmetry elements. *Miner. pol.* 6, 1, 83—89.

Józef NEDOMA

UPROSZCZONY ZAPIS OPERACJI SYMETRII

Streszczenie

Zaproponowano nowy uproszczony zapis operacji symetrii. Uproszczone symbole wszystkich punktowych operacji symetrii spotykanych w krystalografii można z łatwością *przetłumaczyć* na odpowiednie macierze opierając się na macierzy uogólnionej opublikowanej w poprzedniej pracy. Znane operacje symetrii punktowej przedyskutowano w świetle symboli uproszczonego zapisu.

Юзеф НЕДОМА

СОКРАЩЕННАЯ ЗАПИСЬ ОПЕРАЦИИ СИММЕТРИИ

Резюме

Предлагается новая сокращенная запись операций симметрии. Сокращенные символи всех операций точечной симметрии встречаемых в кристаллографии легко *переводятся* в соответствующие матрицы при помощи обобщенной матрицы опубликованной в предыдущей работе. Все известные операции точечной симметрии рассматриваются с точки зрения сокращенной записи.

MINERALOGIA POLONICA
Vol. 8, No 2 — 1977

*Józef NEDOMA**, *Jadwiga POBOŻNIAK*

COEXISTENCE OF SYMMETRY OPERATIONS FROM THE VIEW-POINT OF SIMPLIFIED MATRIX NATION
PART I

UKD 548.12:512.831:003.62

Abstract. Groups of symmetry operations corresponding to simple rotation and inversion axes in terms of simplified matrix symbols are derived.

INTRODUCTION

Each point symmetry operation can be represented by its simplified symbol introduced previously $n\,(MNP)$ for ordinary rotation axes and $\bar{n}\,(MNP)$ for inversion axes. A simple symmetry operation which transforms the coordinates of a point in space x, y, z into corresponding coordinates x', y', z' can transform the resulting coordinates once more into x'', y'', z''. Each symmetry operation can thus coexist with itself. In this paper we will discuss this coexistence in terms of simplified matrix notation. For sake of convenience we will choose the system of coordinates in such a way that the discussed rotation and inversion axes will be described by general symbols $n\,(OOP)$ and $\bar{n}\,(OOP)$ respectively, where $P^2 = 1$. To shorten the notation we will write in these symbols $\bar{1}$ instead of -1.

Operation $6\,(001)$

With aid of the generalized matrix (introducing $\alpha = 60°$, $D = 1$, $M = 0$, $N = 0$, $P = 1$) we can write:

* Academy of Mining and Metallurgy, Institute of Material Science, Cracow (Kraków, al. Mickiewicza 30).

$$6\,(001) = \left\| \begin{matrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{matrix} \right\|$$

Multiplying this matrix by itself we obtain:

$$\left\| \begin{matrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{matrix} \right\|$$

or — after decipheration — $3\,(001)$.

In terms of simplified symbols we can write therefore:

$$6\,(001) \cdot 6\,(001) = 3\,(001)$$

Multiplication $3\,(001) \cdot 6\,(001)$ leads to the matrix $2\,(001)$. Multiplication in reversed order gives the same result:

$$6\,(001) \cdot 3\,(001) = 2\,(001)$$

Proceeding in the same way i.e. multiplying each of new resulting operations by $6\,(001)$ and by all matrices already derived we obtain the following multiplication table:

	$6\,(001)$	$3\,(001)$	$2\,(001)$	$3\,(00\bar{1})$	$6\,(00\bar{1})$
$6\,(001)$	$3\,(001)$	$2\,(001)$	$3\,(00\bar{1})$	$6\,(00\bar{1})$	$1\,(MNP)$
$3\,(001)$	$2\,(001)$	$3\,(00\bar{1})$	$6\,(00\bar{1})$	$1\,(MNP)$	$6\,(001)$
$2\,(001)$	$3\,(00\bar{1})$	$6\,(00\bar{1})$	$1\,(MNP)$	$6\,(001)$	$3\,(001)$
$3\,(00\bar{1})$	$6\,(00\bar{1})$	$1\,(MNP)$	$6\,(001)$	$3\,(001)$	$2\,(001)$
$6\,(00\bar{1})$	$1\,(MNP)$	$6\,(001)$	$3\,(001)$	$2\,(001)$	$3\,(001)$

The operation $6\,(001)$ coexisting with itself generates thus the following symmetry operations:

$$3\,(001) = \left\| \begin{matrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{matrix} \right\|$$

$$2\,(001) = \left\| \begin{matrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{matrix} \right\|$$

$$3\,(00\bar{1}) = \left\| \begin{matrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{matrix} \right\|$$

$$6\,(00\bar{1}) = \left\| \begin{matrix} \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{matrix} \right\|$$

Let us introduce the following symbol

$$\{n_1\,(M_1N_1P_1) \cdot n_2\,(M_2N_2P_2)\}$$

describing all symmetry operations derived from two coexisting symmetry operation written in brackets. For the coexistence $6\,(001) \cdot 6\,(001)$ we can write:

$$\{6\,(001) \cdot 6\,(001)\} = 6\,(001),\ 3\,(001),\ 2\,(001),\ 3\,(00\bar{1}),\ 6\,(00\bar{1}),\ 1\,(MNP)$$

O p e r a t i o n $4\,(001)$

Traducing the simplified symbol of this operation into the corresponding matrix we obtain:

$$4\,(001) = \left\| \begin{matrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{matrix} \right\|$$

Multiplication of this matrix by itself yields:

$$\left\| \begin{matrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{matrix} \right\| = 2\,(001)$$

We can write multiplication as follows:

$$4\,(001) \cdot 4\,(001) = 2\,(001)$$

Continuing the multiplication as in the case of the $6\,(001)$ — operation we obtain the following multiplication table:

	$4\,(001)$	$2\,(001)$	$4\,(00\bar{1})$
$4\,(001)$	$2\,(001)$	$4\,(00\bar{1})$	$1\,(MNP)$
$2\,(001)$	$4\,(00\bar{1})$	$1\,(MNP)$	$4\,(001)$
$4\,(00\bar{1})$	$1\,(MNP)$	$4\,(001)$	$2\,(001)$

where:

$$4\,(00\bar{1}) = \begin{Vmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{Vmatrix}$$

For the operation 4 (001) coexisting with itself we can write:

$$\{4\,(001)\cdot 4\,(001)\} = 4\,(001),\ 2\,(001),\ 4\,(00\bar{1}),\ 1\,(MNP)$$

Operation 3 (001)

To derive all operations resulting from the coexistence $3\,(001)\cdot 3\,(001)$ we can make use directly of the multiplication table for 6 (001).

	3 (001)	3 (00$\bar{1}$)
3 (001)	3 (00$\bar{1}$)	1 (MNP)
3 (00$\bar{1}$)	1 (MNP)	3 (001)

We have thus:

$$\{3\,(001)\cdot 3\,(001)\} = 3\,(001),\ 3\,(00\bar{1}),\ 1\,(MNP)$$

Operation 2 (001)

Multiplication $2\,(001)\cdot 2\,(001)$ leads directly to the operation of identity. We can write:

$$\{2\,(001)\cdot 2\,(001)\} = 2\,(001),\ 1\,(MNP)$$

Operation 1 (MNP)

$$1\,(MNP)\cdot 1\,(MNP) = 1\,(MNP)$$

No new operations are obtained.

$$\{1\,(MNP)\cdot 1\,(MNP)\} = 1\,(MNP)$$

Operation $\bar{6}$ (001)

As demonstrated previously

$$\bar{n}\,(MNP) = [n\,(MNP)\cdot \bar{1}\,(MNP)]$$

The brackets on the right side of this equation account for the fact that the operation $n\,(MNP)$ and $\bar{1}\,(MNP)$ do not exist separately. As a symmetry

operation acts only the product of these both operations. In the case of coexistence of two inversion axes we can write:

$$\bar{n}_1\,(M_1N_1P_1)\cdot \bar{n}_2\,(M_2N_2P_2) = [n_1\,(M_1N_1P_1)\cdot \bar{1}\,(MNP)\cdot n_2\,(M_2N_2P_2)\cdot \bar{1}\,(MNP)]$$

Besing on the fact that

$$n\,(MNP)\cdot \bar{1}\,(MNP) = \bar{1}\,(MNP)\cdot n\,(MNP)$$

and

$$\bar{1}\,(MNP)\cdot \bar{1}\,(MNP) = 1\,(MNP)$$

we can rewrite this equation

$$\bar{n}_1\,(M_1N_1P_1)\cdot \bar{n}_2\,(M_2N_2P_2) = n_1\,(M_1N_1P_1)\cdot n_2\,(M_2N_2P_2)$$

For the case of coexistence $\bar{n}_1\,(M_1N_1P_1)$ and $n_2\,(M_2N_2P_2)$ we obtain

$$\bar{n}_1\,(M_1N_1P_1)\cdot n_2\,(M_2N_2P_2) = [n_1\,(M_1N_1P_1)\cdot n_2\,(M_2N_2P_2)\cdot \bar{1}\,(MNP)]$$

To derive all operation resulting from coexistence $\bar{6}\,(001)$ with itself we can make use of the multiplication table derived for 6 (001):

	$\bar{6}$ (001)	3 (001)	$\bar{2}$ (001)	3 (00$\bar{1}$)	$\bar{6}$ (00$\bar{1}$)
$\bar{6}$ (001)	3 (001)	$\bar{2}$ (001)	3 (00$\bar{1}$)	$\bar{6}$ (00$\bar{1}$)	1 (MNP)
3 (001)	$\bar{2}$ (001)	3 (00$\bar{1}$)	$\bar{6}$ (00$\bar{1}$)	1 (MNP)	$\bar{6}$ (001)
$\bar{2}$ (001)	3 (00$\bar{1}$)	$\bar{6}$ (00$\bar{1}$)	1 (MNP)	$\bar{6}$ (001)	3 (001)
3 (00$\bar{1}$)	6 (00$\bar{1}$)	1 (MNP)	$\bar{6}$ (001)	3 (001)	$\bar{2}$ (001)
$\bar{6}$ (00$\bar{1}$)	1 (MNP)	$\bar{6}$ (001)	3 (001)	$\bar{2}$ (001)	3 (00$\bar{1}$)

where:

$$\bar{6}\,(001) = \begin{Vmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & -1 \end{Vmatrix}$$

$$\bar{2}\,(001) = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{Vmatrix}$$

$$\bar{6}\,(00\bar{1}) = \begin{Vmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & -1 \end{Vmatrix}$$

We can write:

$$\{\bar{6}\,(001)\cdot\bar{6}\,(001)\} = \bar{6}\,(001),\ 3\,(001),\ \bar{2}\,(001),\ 3\,(00\bar{1}),\ \bar{6}\,(00\bar{1}),\ 1\,(MNP)$$

Operation $\bar{4}\,(001)$

Using the multiplication table for 4 (*001*) we obtain directly:

	$\bar{4}\,(001)$	$2\,(001)$	$\bar{4}\,(00\bar{1})$
$\bar{4}\,(001)$	$2\,(001)$	$\bar{4}\,(00\bar{1})$	$1\,(MNP)$
$2\,(001)$	$\bar{4}\,(00\bar{1})$	$1\,(MNP)$	$\bar{4}\,(001)$
$\bar{4}\,(00\bar{1})$	$1\,(MNP)$	$\bar{4}\,(001)$	$2\,(001)$

where:

$$\bar{4}\,(001) = \begin{Vmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{Vmatrix} \qquad \bar{4}\,(00\bar{1}) = \begin{Vmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{Vmatrix}$$

For $\bar{4}\,(001)\cdot\bar{4}\,(001)$ we can thus write:

$$\{\bar{4}\,(001)\cdot\bar{4}\,(001)\} = \bar{4}\,(001),\ \bar{4}\,(00\bar{1}),\ 2\,(001),\ 1\,(MNP)$$

Operation $\bar{3}\,(001)$

Using the multiplication table derived for 6 (*001*) we obtain

	$\bar{3}\,(001)$	$3\,(00\bar{1})$	$\bar{1}\,(MNP)$	$3\,(001)$	$\bar{3}\,(00\bar{1})$
$\bar{3}\,(001)$	$3\,(00\bar{1})$	$\bar{1}\,(MNP)$	$3\,(001)$	$\bar{3}\,(00\bar{1})$	$1\,(MNP)$
$3\,(00\bar{1})$	$\bar{1}\,(MNP)$	$3\,(001)$	$\bar{3}\,(00\bar{1})$	$1\,(MNP)$	$\bar{3}\,(001)$
$\bar{1}\,(MNP)$	$3\,(001)$	$\bar{3}\,(00\bar{1})$	$1\,(MNP)$	$\bar{3}\,(001)$	$3\,(00\bar{1})$
$3\,(001)$	$\bar{3}\,(00\bar{1})$	$1\,(MNP)$	$\bar{3}\,(001)$	$3\,(00\bar{1})$	$\bar{1}\,(MNP)$

where:

$$\bar{3}\,(001) = \begin{Vmatrix} \frac{1}{2} & \frac{3}{2} & 0 \\ -\frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 0 & -1 \end{Vmatrix}$$

$$\bar{3}\,(00\bar{1}) = \begin{Vmatrix} \frac{1}{2} & -\frac{3}{2} & 0 \\ \frac{3}{2} & \frac{1}{2} & 0 \\ 0 & 0 & -1 \end{Vmatrix}$$

We can therefore write

$$\{\bar{3}\,(001)\cdot\bar{3}\,(001)\} = \bar{3}\,(001),\ \bar{3}\,(00\bar{1}),\ 3\,(001),\ 3\,(00\bar{1}),\ \bar{1}\,(MNP),\ 1\,(MNP)$$

Operation $\bar{2}\,(001)$

Multiplication $\bar{2}\,(001)\cdot\bar{2}\,(001)$ yields 1 (*MNP*). We have therefore

$$\{\bar{2}\,(001)\cdot\bar{2}\,(001)\} = \bar{2}\,(001),\ 1\,(MNP)$$

Operation $\bar{1}\,(MNP)$

After multiplication we obtain immediately the operation of identity.

CONCLUSIONS

The results obtained above can be summarized as follows:

$$\{6\,(001)\cdot 6\,(001)\} = 6\,(001),\ 3\,(001),\ 2\,(001)$$
$$6\,(00\bar{1}),\ 3\,(00\bar{1}),\ 1\,(MNP)$$

$$\{4\,(001)\cdot 4\,(001)\} = 4\,(001),\ 2\,(001)$$
$$4\,(00\bar{1}),\ 1\,(MNP)$$

$$\{3\,(001)\cdot 3\,(001)\} = 3\,(001),\ 1\,(MNP)$$
$$3\,(00\bar{1})$$

$$\{2\,(001)\cdot 2\,(001)\} = 2\,(001),\ 1\,(MNP)$$

$$\{1\,(MNP)\cdot 1\,(MNP)\} = 1\,(MNP)$$

$$\{\bar{6}\,(001)\cdot\bar{6}\,(001)\} = \bar{6}\,(001),\ 3\,(001),\ \bar{2}\,(001)$$
$$\bar{6}\,(00\bar{1}),\ 3\,(00\bar{1}),\ 1\,(MNP)$$

$$\{\bar{4}\,(001)\cdot\bar{4}\,(001)\} = \bar{4}\,(001),\ 2\,(001)$$
$$\bar{4}\,(00\bar{1}),\ 1\,(MNP)$$

$$\{\bar{3}\,(001)\cdot\bar{3}\,(001)\} = \bar{3}\,(001),\ 3\,(001),\ \bar{1}\,(MNP)$$
$$\bar{3}\,(00\bar{1}),\ 3\,(00\bar{1}),\ 1\,(MNP)$$

$$\{\bar{2}\,(001)\cdot\bar{2}\,(001)\} = \bar{2}\,(001),\ 1\,(MNP)$$

$$\{\bar{1}\,(MNP)\cdot\bar{1}\,(MNP)\} = \bar{1}\,(MNP),\ 1\,(MNP)$$

Józef NEDOMA, *Jadwiga* POBOŻNIAK

WSPÓŁISTNIENIE OPERACJI SYMETRII W ŚWIETLE UPROSZCZONEGO ZAPISU MACIERZOWEGO CZĘŚĆ I

Streszczenie

Wyprowadzono grupy operacji symetrii odpowiadających zwykłym i inwersyjnym osiom symetrii posługując się symbolami uproszczonego zapisu macierzowego.

Юзеф НЭДОМА, *Jadwiga* POBOŻNIAK

СОСУЩЕСТВОВАНИЕ ОПЕРАЦИИ СИММЕТРИИ С ТОЧКИ ЗРЕНИЯ СОКРАЩЕННОЙ ЗАПИСИ ЧАСТЬ I

Резюме

Выведены группы операции симметрии соответствующих обычным и инверсионным осям симметрии применяя симболы сокращенной матричной записи.